Bholanath Kumbhakar

Curriculum Vitae

Purulia, West Bengal, India, 723148
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Education

Current Status — **Postdoc student**

- **Institute:** Indian Institute of Science Education and Research Thiruvananthapuram, Kerala, India
- **Research Area:** Optimal Control of Cahn-Hilliard Equation
- **Supervisor:** Prof. Sheetal Dharmatti

December 2018-February 2025 — **Ph. D. in Mathematics**

- **Award date:** February 25, 2025
- **Institute:** Indian Institute of Technology Roorkee
- **Title of Thesis:** Controllability of Some Semilinear Differential Inclusions and Hemivariational Inequalities
- **Research Area:** Differential Inclusion, Partial Differential Equations, Fluid Dynamics, Stochastic Differential Equation
- **Supervisor:** Prof. Dwijendra Narain Pandey

July 2016-May 2018 — **M. Sc. in Mathematics**, CGPA: 8.38 The University of Burdwan, Burdwan, West Bengal, India

July 2013-May 2016 — **B.Sc. in Mathematics**, Percentage: 79.875%, Jagannath Kishore College, Purulia, West Bengal, India

2013 — **Higher Secondary in Science**, Percentage: 77.2%, Purulia Jilla School, Purulia, West Bengal, India

2011 — **Class 10th**, Percentage: 81.875%, Ladhurka High School (H.S), Purulia, West Bengal, India

List of Publications

- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "Approximate controllability of evolution hemivariational inequalities in Banach spaces", Journal of Differential Equations 410 (2024): 346-381.
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "Approximate controllability of nonconvex valued semilinear differential inclusion", Evolution Equations and Control Theory 14.1 (2025): 96-122, doi: 10.3934/eect.2024047
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "L^p- null controllability of abstract differential inclusion with the nonlocal condition", Mathematical Control and Related Fields, 2025, 15(3): 1121-1149. doi: 10.3934/mcrf.2025001.
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "Exact Controllability of Hemivariational Inequalities in Banach spaces", Applied Mathematics & Optimization 92.1 (2025): 18.

- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "L^p- Exact controllability of abstract differential inclusion with nonlocal condition", (Accepted for publication in the Journal Topological Methods in Nonlinear Analysis.)
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "Approximate controllability of semilinear differential inclusion with nonlocal conditions", 2023 Proceedings of the Conference on Control and its Applications (CT). Society for Industrial and Applied Mathematics, 2023.

List of Preprints

- Kumbhakar, Bholanath, Deeksha and Dwijendra Narain Pandey, Approximate Controllability of Stochastic Hemivariational Control problem in Hilbert spaces, https://doi.org/10.48550/arXiv.2504.13954
- Kumbhakar, Bholanath, Deeksha and Dwijendra Narain Pandey, Approximate Controllability of Stochastic Hemivariational Control problem in Banach spaces (Under Preparation)
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, Existence of Solutions of Nonconvex Multivalued Navier Stokes Equations, https://doi.org/10.48550/arXiv.2503.08351
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, Existence of Solutions of Nonconvex Multivalued Version of Oldroyd Model (under preparation)
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, Approximate Controllability of Nonconvex valued Semilinear Differential Inclusions with Nonlocal Conditions, (Submitted)
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey."Approximate Controllability of Fractional Hemivariational Inequalities in Banach Spaces." arXiv preprint arXiv:2408.05536 (2024).
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey. "Optimal Control and Approximate Controllability of Fractional Semilinear Differential Inclusion Involving ψ-Hilfer Fractional Derivatives." arXiv preprint arXiv:2306.01352 (2023).
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, "Approximate Controllability of Semilinear Differential Inclusions with Mixed Semicontinuous Right Hand Side", (Submitted)

Research Interest

- **Differential Inclusion**
  Differential inclusions constitute an active branch of the general theory of differential equations. Differential inclusions, as opposed to ordinary differential equations, encompass a broader range of possible trajectories, representing phenomena involving uncertain inputs, discontinuities, and various types of constraints. Because of this adaptability, differential inclusions are particularly effective at capturing situations in which the evolution of a system is subject to ambiguous or non-deterministic influences. As a result, they are used in various fields, including control theory, economics, ecology, and engineering, where uncertainty, variability, and discontinuities are common.

- **Control Theory**
  The controllability problem may be formulated roughly as follows. Consider an evolution system either described in terms of partial differential equations (PDEs) or ordinary differential equations (ODEs) on which we are allowed to act by means of a suitable choice of control (the right-hand side of the system, the boundary conditions, etc.). Given a time interval $I := [0, a]$ and initial and final states, the goal is to determine whether there exists a control driving the given initial data to the given final ones in time a. If the system can be steered from any given initial state to any given final state in finite time using control input(s, then we say that the system is exactly controllable. If the system can be steered from any given initial state to the origin, then we say the system is null controllable. On the other hand, approximate controllability means we can steer the state of a system from any given initial state to arbitrarily close to a desired final state using control input(s) rather than reaching the exact final state.

- **Hemivariational Inequality**
  Hemivariational inequalities were first introduced by Panagiotopoulos in the early 1980s and are closely linked to Clarke's development of the generalized gradient of a locally Lipschitz functional. Like variational inequalities, the initial interest in hemivariational inequalities emerged from mechanical problems. These inequalities effectively handle problems involving nonmonotone, nonsmooth, and multivalued constitutive laws, forces, and boundary conditions, both theoretically and numerically. Over the past three decades, hemivariational inequalities have proven useful across various fields, leading to numerous problems modeled mathematically in terms of these inequalities.

- **Fractional Differential Equations**
  It is worth noting that most existing research works focused on integer order dynamics. However, in the real world, it has been shown that many natural phenomena cannot be effectively interpreted by the integer order dynamics, such as chemotaxis behavior and food searching of germs. Nevertheless, fractional order dynamics possess excellent memory and hereditary properties, resulting in superior performance and stronger robustness than standard integer order dynamic systems. It has been proved that some cases, such as the macromolecule fluids, lateral inhibition of biological vision systems, and automobiles running on the road's surface containing viscoelastic materials, can be more accurately described by fractional-order dynamic systems. Researchers pointed out that many physical systems are unsuitable to be characterized by integer order dynamics, such as high-speed aircraft traveling on rainy days or snowy days and vehicles moving on top of sand or muddy road.

- **Stochastic Differential Equation**
  Many real-world phenomena—such as population dynamics, stock market fluctuations, weather forecasting models, and heat conduction in materials with memory—are influenced by inherent randomness. Due to the presence of noise, deterministic models often undergo significant modifications. In fact, stochastic disturbances are not only unavoidable but also widespread in both natural and engineered systems. Consequently, deterministic models frequently fail to capture the true nature of fluctuations observed in practice. To better reflect reality, it becomes essential to incorporate stochastic processes into the modeling framework. In numerous cases, either the initial data and system parameters are randomly perturbed, or the underlying dynamics are inherently stochastic. This necessitates the integration of stochastic effects into the study of differential systems, leading naturally to the formulation and analysis of stochastic evolution equations.

Brief summary of Research Work

- Differential inclusions, which naturally extend the concept of ordinary differential equations, have become integral part to various scientific fields due to their wide-ranging applications, forming a significant branch within the general theory of differential equations. In addressing the existence of mild solutions and the controllability of differential inclusions, both topological properties (such as continuity, measurability, and compactness) and geometric properties (such as convexity) of the associated multivalued maps are crucial. Notably, the results differ substantially between convex and nonconvex valued multimaps, as do the continuity properties, including upper semicontinuity, lower semicontinuity, and Hausdorff continuity. Additionally, the characteristics of the state and control spaces are pivotal in determining the existence and controllability of solutions. Research conducted in Banach spaces presents more significant challenges compared to Hilbert spaces. With these considerations in mind, we focused on the controllability of systems governed by semilinear differential inclusions and hemivariational inequalities, specifically examining exact, approximate, and null controllability problems.

- We observe that many models in mechanics, engineering, chemistry, and fluid flow have recently been successfully modeled by fractional hemivariational inequalities. The constitutive equation for a quasistatic frictionless contact problem of a viscoelastic body is modeled with the fractional Kelvin-Voigt law, and the contact condition is described by the Clarke subdifferential of a nonconvex and nonsmooth functional. The variational formulation of this problem is provided in the form of a fractional hemivariational inequality. Therefore, we study the exact and approximate controllability of systems governed by evolution fractional hemivariational inequalities involving Caputo fractional derivatives. We also established the approximate controllability and optimal controllability of systems involving ψ-Hilfer fractional derivatives.

Honors & Awards

CSIR JRF (December 2017), Got 98th rank in the NET exam conducted by CSIR
CSIR JRF (June 2018), Got 98th rank in the NET exam conducted by CSIR

Teaching Assistant

Spring 2022 **Mathematical Methods**, B. Tech. 1st Year, IIT Roorkee
- **Tutorial Topic:** Ordinary Differential Equations, Partial Differential Equations, Laplace Transform, Z-Transform, Fourier Series, Fourier Transforms
- **Instructor:** Prof. Saikat Saha

Autumn 2021 **Mathematics I**, B.Tech. 1st Year, IIT Roorkee
- **Tutorial Topoic:** Matrix Algebra, Differential Calculus, Integral Calculus, Vector Calculus
- **Instructor:** Prof. Ankik Kumar Giri

Spring 2021 **Mathematical Methods**, B.Tech 1st Year, IIT Roorkee
- **Instructor:** Prof. Ram Jiwari

Autumn 2020 **Mathematics I**, 1st year B. Tech., IIT Roorkee
- **Instructor:** Prof. S. K. Gupta

Spring 2020 **Mathematics I**, B. Tech. 1st Year, IIT Roorkee
- **Instructor:** Prof. Rama Bhargava

Workshops

- NCMW - Control Theory for Differential Equations (2022)
  Date: November 28- December 10, 2022
  Venue: IISER Kolkata
- International Workshop on Fractional Derivatives: Theory and Computations with Applications (FDTCA 2021)
  Date: November 12-14, 2021
  Venue: Department of Mathematical Sciences Indian Institute of Technology (BHU) Varanasi
- Control theory meets the theory of Homogenization
  Date: February 28- March 3, 2023
  Venue: Ramanujan Hall, Department of Mathematics, IIT Bombay
- Fundamentals of Differential Equations with Applications to Real World Problems
  Date: July 8- July 13, 2024
  Venue: Department of Mathematics, Jaypee University of Information Technology, Waknaghat (Shimla Hills), Himachal Pradesh, INDIA

Conference

- 87^{th} Annual Conference of the Indian Mathematical Society
  Date: December 4-7, 2021
  Venue: Online
  Orginazed by: GM University, Maharashtra
- International Conference on Dynamical systems, Control and their Applications
  Date: July 1-3, 2022
  Venue: IIT Roorkee
- SIAM Conference on Control Theory and Its Applications (SIAM CT 2023)
  Date: July 24-26, 2023
  Venue: Sonesta Philadelphia Rittenhouse Square Hotel, Philadelphia, Pennsylvania, USA
- The International Congress on Industrial and Applied Mathematics (ICIAM) 2023 (online)
  Date: August 20-25, 2023, Organized by: Waseda University, Tokyo, Japan
- The 14th AIMS Conference on Dynamical Systems, Differential Equations and Its Applications
  Date: December 16-20, 2024
  Organized by: New York University, Abu Dhabi, United Arab Emirates
  Venue: ADNEC Center, Abu Dhabi

Languages:

Bengali (Native), Hindi, English

References

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Declaration: I hereby declare that all the statements made herein are true to my best of knowledge and belief.

Date: 11-05-2025 Bholanath Kumbhakar